

# Function Composition with Dynagraphs

In life, the answer to one question sometimes becomes a question that leads to another answer. Functions are much the same; sometimes we take the output of one function and make it the input for a second function. This is called *function composition*, and we say that the two functions have been *composed*. In this activity you'll get a brief introduction to function composition and then see how dynagraphs can provide a compelling way of modeling composed functions.

## INTRODUCTION

We'll introduce function composition informally by doing some examples with numbers.

The composite function  $g(f(3))$  is pronounced "g of f of 3."

Given  $f(x) = 2x$  and  $g(x) = x^2$ , find  $g(f(3))$ .

You always evaluate parentheses first, so start on the inside by evaluating  $f(3)$ :  $f(3) = 2 \cdot (3) = 6$ .

Take this output and make it  $g$ 's input:  $g(f(3)) = g(6) = (6)^2 = 36$ .

That, in a tiny nutshell, is function composition.

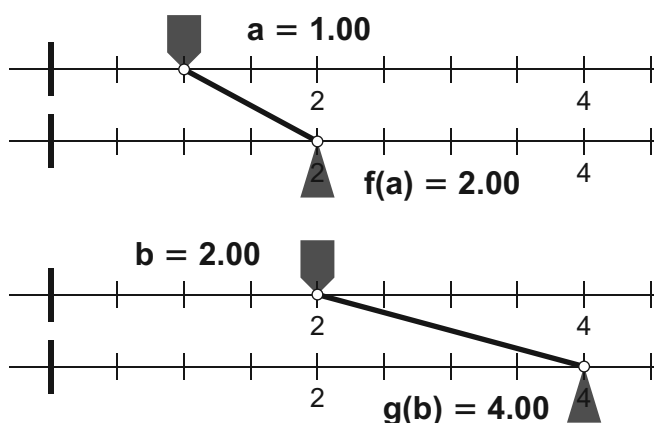
**Q1** Given the functions

$$f(x) = 2x, g(x) = x^2, h(x) = \text{round}(x), \text{ and } j(x) = \frac{x}{2},$$

evaluate the following expressions:

- |               |               |                |
|---------------|---------------|----------------|
| a. $g(f(-1))$ | b. $f(g(3))$  | c. $f(h(3.6))$ |
| d. $j(g(-6))$ | e. $j(f(17))$ | f. $f(j(17))$  |

**Q2** Do you think  $f(g(x))$  always equals  $g(f(x))$ ? Answer this question by comparing  $f(g(5))$  and  $g(f(5))$ .



## SKETCH AND INVESTIGATE

1. Open **Composite Functions.gsp**. Drag each input marker to familiarize yourself with the dynagraphs of functions  $f$  and  $g$ .

You'll use the dynagraphs to find  $g(f(-1))$ .

**Q3** Start by finding  $f(-1)$ . Drag the input marker for  $f$  to  $-1$ . What's the value of the output marker?

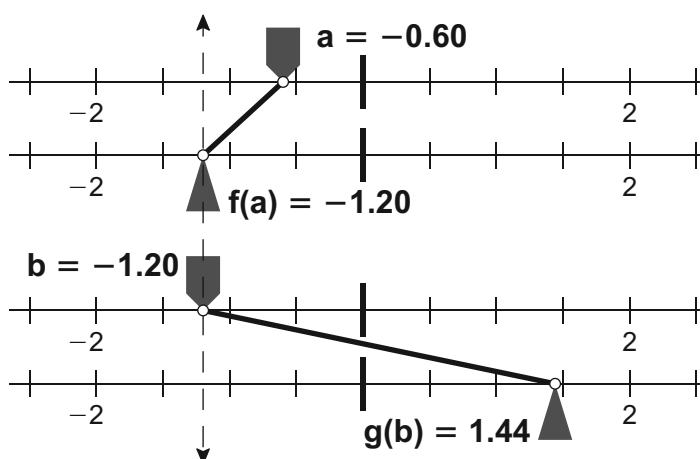
**Q4** Now find  $g(f(-1))$ , by dragging the input marker for  $g$  to match the output marker for  $f$ . What's the value of  $g$ 's output marker? Does this match your answer from Q1 part a?

**Q5** Now use the dynagraphs to find  $g(f(1.20))$ . You'll need to adjust  $f$ 's input to  $1.20$  and then adjust  $g$ 's input to match  $f$ 's output. What result do you get?

To evaluate  $g(f(x))$  more easily, you need to make  $g$ 's input marker automatically match  $f$ 's output marker.

2. To transfer the output from  $f$  to the input for  $g$ , you'll construct a vertical line. Select point  $f(a)$  (at the tip of  $f$ 's output marker) and the output axis for  $f$ . Then choose **Construct | Perpendicular Line**.
3. Use the **Point** tool to construct the intersection of the vertical line with the input axis for  $g$ .
4. Select point  $b$  (at the tip of  $g$ 's input marker) and choose **Edit | Split Point from Line**. The point is separated from its axis.
5. Select points  $b$  and the intersection point. Choose **Edit | Merge Points**.

The two points are merged into one, and the output of  $f$  is the input of  $g$ . Drag the input marker of  $f$  to explore your new composite function,  $g(f(x))$ , and to check your answer from Q1 part a.



6. Go to page "g&f", which contains the same two functions, but with a  $g$  above and  $f$  below. Use the method from steps 4 and 5 to model  $f(g(x))$ .

**Q6** Use your composite dynagraph to evaluate these expressions:

- a.  $f(g(1))$                       b.  $f(g(-1))$                       c.  $f(g(-7))$

Next, you'll compose the "round" and "square" functions to create a composite function with an interesting set of outputs.

**Q7** Go to page "h&g" and model  $g(h(x))$ . What is the range of this composite function? In other words, what are its possible outputs?

**Q8** Create the composite functions  $j(f(x))$  and  $f(j(x))$  on the appropriate pages. Experiment with these functions. What special feature do you notice about these two composite functions? Why does this happen?

Use the remaining pages of the document to experiment with other composite functions.

## EXPLORE MORE

**Q9.** There's no reason you can't compose more than two functions to get something such as  $h(g(f(x)))$ . Go to page "all 4" and try this for different combinations of three or four functions. See if you can build the following functions:

- a function that outputs twice perfect squares (0, 2, 8, 18, ...)
- a function that outputs squares of even numbers (0, 4, 16, 36, ...)
- a function that outputs the perfect squares divided by 4 (0, 0.25, 1, 2.25, 4, 6.25, ...)