## Create a Dynagraph

In this activity you will compose a function on a number line, as you have already done. But this time you will display the dependent variable on its own number line to make it easier to compare the motion of the two variables. We will call the two number lines the input axis and the output axis.

## DILATE

Begin by constructing the input axis.

1. Open http://geometricfunctions.org and navigate to Activities | Cartesian Connection | Create a Dynagraph. Go to page 2 of the sketch.
2. Construct a Number Line ${ }^{5}$ $\square$ on the line, and
 point $x$. Adjust the number line to make it horizontal.

Q1 Each line in this table describes the relative motion of $x$ and $D(x)$. Find a scale factor that creates the given motion, and write it down. Check your result by varying $x$.

| Relative speed | Relative direction | Scale factor $\boldsymbol{s}$ |
| :--- | :--- | :---: |
| $D(x)$ goes slower than $x$. | $D(x)$ goes the opposite direction as $x$. | $\boldsymbol{s}=$ |
| $D(x)$ goes faster than $x$. | $D(x)$ goes the same direction as $x$. | $\boldsymbol{s}=$ |
| $D(x)$ goes the same speed as $x$. | $D(x)$ goes the opposite direction as $x$. | $\boldsymbol{s}=$ |
| $D(x)$ goes slower than $x$. | $D(x)$ goes the same direction as $x$. | $\boldsymbol{s}=$ |

## CREATE THE OUTPUT AXIS


tool. Attach glowing point 0 to point 0 of the input axis, and attach glowing $D(x)$ to $D(x)$ on the input axis.
4. Use the Number Line tool. Attach the new number line to the lower orange point and make it horizontal. This new number line will be the output axis.
5. You should be able to drag the origin of the output axis up or down, but not sideways. Also, the lower $D(x)$ point should always stay on the output axis.

## TRANSLATE

Use the output axis to translate $D(x)$.
6. Use the $\underset{\text { Translate }}{\bullet}{ }^{\bullet} D(x)$ tool to translate $D(x)$ (on the output axis) by vector $v$. Be sure to attach point $v$ to the axis. The result, $T(D(x))$, is "the translation of the dilation of $x$." Drag point $v$ back and forth to observe the result of the translation.

## INVESTIGATE

Q2 For each row of the table, edit $s$ to the value shown, and drag points $v$ and $x$ so they have the indicated values. Then record the values for $D(x)$ and $T(D(x))$.

| $\boldsymbol{S}$ | $\boldsymbol{v}$ | $\boldsymbol{x}$ | $\boldsymbol{D}(\boldsymbol{x})$ | $\boldsymbol{T}(\boldsymbol{D}(\boldsymbol{x}))$ |
| :---: | :---: | :---: | :---: | :---: |
| 2.50 | -3.00 | -2.00 |  |  |
| -2.00 | 1.00 | -3.00 |  |  |
| 0.50 | 2.50 | 2.00 |  |  |

7. Press the Hide Transfer and Hide Dilation buttons.
8. Use the Connect to to connect $x$ with $T(D(x))$, and turn tracing on. Experiment using different scale factors to see what the traces of the connecting line look like.

Q3 For $\boldsymbol{a}$ below, set the dilation scale factor ( $s$ ) and the translation vector ( $v$ ) as shown. Then vary $x$ and draw in some of the connecting lines that are traced. Then do $\mathbf{b}$ and $\mathbf{c}$.


Q4 For the traces on the left, circle the correct words on the right, and tell how you know.

|  | $T(D(x))$ is moving \| faster | slower | at the same speed | as $x$. <br> $T(D(x))$ is moving in the \| same | opposite | direction as $x$. <br> I know this because: |
| :--- | :--- |
|  | $T(D(x))$ is moving \| faster | slower | at the same speed | as $x$. <br> $T(D(x))$ is moving in the \| same | opposite | direction as $x$. <br> $I$ know this because: |

## SOLVE A MYSTERY

9. On page 3 create another dynagraph just as you did on page 2 .
10. Use the Animate tool to create an Animate x button, and use the Mystery create a mystery function, labeled ??(x).
11. For your first mystery function, start with level 1 . To solve the mystery, adjust $s$ and $v$ to make $T(D(x))$ line up exactly with the mystery function.

You'll know you have it right when you can press Animate x and see $T(D(x))$ and ??(x) lined up exactly all the way across the screen. Once you have solved three or four mystery functions answer the question below.

Q5 If a friend asks for your help in solving mystery functions, what hints and suggestions would you give her?
$\square$

## PLAY THE MYSTERY GAME

12. Tap the link beneath the sketch to "Play the Dynagraph Game." Start on Level 1, but with practice you should be able to master Level 3 or, even better, Level 4.
