

In this activity you'll create two symbolically-defined Sketchpad functions and use them to calculate output values of the composed function. You'll then create a third function that's the composition of the first two, allowing you to do the two steps of the calculation in a single step.

CREATE AND EVALUATE TWO FUNCTIONS

Sketchpad labels it $f(x)$.

Sketchpad labels it $g(x)$.

Sketchpad labels the result $f(3.5)$.

Sketchpad labels the result $g(f(3.5))$.

1. Open **Symbolic Composition.gsp**, and go to page 1, which is empty.
2. Create a “2 times” function to be function f . [Choose **Number** | **New Function**, type “ $2x$ ”, and press OK.]

3. Create an “add 3” function to be function g . [Choose **Number** | **New Function**, type “ $x + 3$ ”, and press OK.]

Q1 If you apply function f to 3.5, what number will you get? If you apply function g to the result from applying f , what number will you get?

4. Apply function f to 3.5. [Choose **Number** | **Calculate**, click $f(x)$ in the sketch, type 3.5, and press OK.]

5. Apply function g to the result from step 4. [Choose **Number** | **Calculate**, click $g(x)$ in the sketch, click $f(3.5)$ in the sketch, and press OK.]

Q2 Does the result agree with your prediction in Q1?

$$f(x) = 2 \cdot x$$

Q3 Explain why the result's label, $g(f(3.5))$, makes sense. (Refer to the operations that f and g stand for.)

$$g(x) = x + 3$$

$$f(3.5) = 7.00$$

$$g(f(3.5)) = 10.00$$

To easily use different numbers, make the calculation use a variable instead of the fixed value 3.5.

6. Edit the $f(3.5)$ calculation from step 4 to replace the fixed value 3.5 with a variable z . [Double-click the calculation to edit it, select the 3.5 in the Calculator pane, and choose **New Parameter** from the Calculator's Values menu. In the New Parameter dialog box set the name to z , set the value to 3.5, and press OK. Then press OK in the Edit Calculation dialog box.]

Q4 With a selected, press the $+$ sign on the keyboard several times. Then press the $-$ sign and hold it down to change the value to 1.5. What happens when you press the $+$ and $-$ keys?

$$f(x) = 2 \cdot x$$

$$g(x) = x + 3$$

$$f(z) = 3.00$$

$$g(f(z)) = 6.00$$

Q5 Change z to -1.0 . What is the value of $f(z)$? What's the value of $g(f(z))$?

$$z = 1.5$$

Vary z Discretely

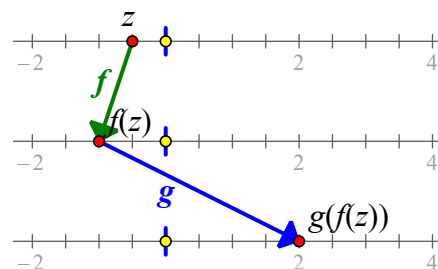
For more details, use **Help** | **Reference Center** to look up “keyboard adjustment.”

Instead of using $+$ and $-$ keys, you can edit a directly.

7. Make a button to animate z between -5 and 5 discretely by 0.2 unit per 0.5 second. Label it *Vary z Discretely*. Then press the button. Press it again to stop. [With z selected, choose **Edit** | **Action Buttons** | **Animation** and make the value change discretely by 0.5 units per 1 sec. from -5 to 5 .]

CREATE A COMPOSED DYNAGRAPH

8. Copy all six objects from page 1 and paste them onto page 2.
9. Click the **Number Line** custom tool on the four points on page 2 to make four horizontal axes. (You'll use the bottom one later.)
10. Plot the value of a on the top axis, the value of $f(z)$ on the second axis, and the value of $g(f(z))$ on the third axis. [To plot a value on an axis, select the value and the axis and choose **Graph | Plot Value on Axis.**]
11. Label each point with the name of its variable: z , $f(z)$, and $g(f(z))$.
12. Connect points z and $f(z)$ with a traced arrow f , and connect points $f(z)$ and $g(f(z))$ with a traced arrow g in a different color. [To construct a traced arrow, choose the **Traced Arrow** custom tool.]



- Q6 What function does each traced arrow represent? Explain.
- Q7 Press the action button to animate the variable z . How does the pattern of each set of traces relate to the function it represents?

COMPOSE SYMBOLICALLY

Now you'll create a single *composed* function h that does both steps. In other words, this function will “add 3 to the result of multiplying by 2.”

13. Create a third function: $h(x) = g(f(x))$. [Choose **Number | New Function**, click $g(x)$ in the sketch, click $f(x)$ in the sketch, type “ x ”, and press OK.]
- Q8 In the expression $g(f(x))$, explain which function should actually be applied first, and which should be applied second. Your explanation should make sense to someone who might be confused about the order.
14. Calculate $h(z)$. Plot $h(z)$ on the fourth axis and connect it to $g(f(z))$ on the third axis with a traced segment.
- Q9 What's the value of this calculation? How does it compare to $g(f(z))$?
- Q10 Press the action button, and observe the bottom traced segment. What pattern does it show? Explain what this pattern means.

EXPLORE MORE

- Q11 Edit functions f and g to be different functions of your choice. Then animate again, and again observe the bottom traced segment. What do you observe?