

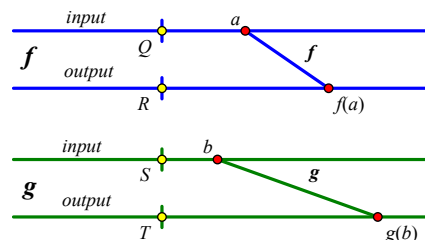
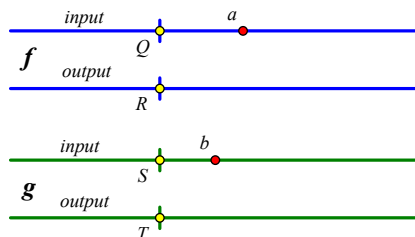
Dynagraphs were invented by Paul Goldenberg, Philip Lewis, and James O'Keefe.

A dynagraph is a function with its domain restricted to a horizontal line (the *input axis*), and its range on another horizontal line (the *output axis*).

In this activity you will create two dynagraph functions, and then compose them by merging the input of one to the output of the other.

## CONSTRUCT TWO DYNAGRAPHS

1. Open the sketch **Dynagraph Function Composition.gsp**. The sketch contains four points with tick marks attached. You will use these points to make four horizontal axes.
2. Construct the axes. [Select the blue *Unit* segment and also points  $Q$ ,  $R$ ,  $S$ , and  $T$ , and then choose **Construct** | **Parallel Lines**.]
3. Construct independent variables  $a$  and  $b$ . [Click the **Point** tool on each input axes, and label the new points  $a$  and  $b$ .]
4. Dilate variable  $a$  about point  $Q$  by a factor of 2. [Select point  $a$  and choose **Transform** | **Dilate**. When the dialog box appears, click point  $Q$  in the sketch to mark it as the center, and change the scale factor to 2.0/1.0.]
5. To construct the dependent variable, transfer the dilated point to the output axis, and connect it to its independent variable. Label the output point  $f(a)$ , and label the connection  $f$ . [Select the dilated point, choose **Transform** | **Transfer to Output**, and hide the dilated point on the *input* axis. Use the **Segment** tool to connect the output point to point  $a$ , and use the **Text** tool to label the output point and the connection.]
6. Translate variable  $b$  by three units to the right. [Click the *Show Distance Ruler* button to show a ruler of units. Then select point  $b$  and choose **Transform** | **Translate**. When the dialog box appears, mark the translation vector on the ruler by clicking point 0 and then point 3.]
7. To construct the dependent variable, transfer the translated point to the output axis, label it  $g(b)$ , and connect it to its independent variable. [Use the same procedure as in step 5 to transfer the translated point, hide the translated point, connect the output point to independent variable  $b$ , and label the dependent point  $g(b)$  and label the connection  $g$ .]



## OBSERVE FUNCTION BEHAVIOR

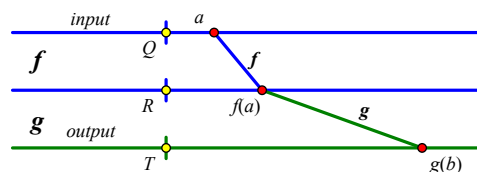
- Q1 Drag independent variable  $a$  and observe the behavior of the dependent variable  $f(a)$ . Compare their speeds and the direction of their motion. Are there any fixed points? (A *fixed point* is a point at which the two dynagraph variables are exactly aligned vertically.)
8. Turn on tracing for segments  $f$  and  $g$ . [Select the segments and choose **Display | Trace Segments.**]
- Q2 Drag  $a$ . What pattern do the traces make? Sketch it on your paper. How does this pattern relate to the relative rate of change of the two variables?
- Q3 Drag  $b$  and observe the behavior of dependent variable  $g(b)$ . Compare the two variables' speeds and directions. Are there any fixed points?
- Q4 What pattern do the traces make? Sketch it on your paper. How does this pattern relate to the relative rate of change of these two variables?

## MERGE THE AXES AND COMPOSE THE FUNCTIONS

To compose the functions, you'll use the output of the first —  $f(a)$  — as the input for the second.

9. To prepare the axes, merge points  $R$  and  $S$ . [Select both  $R$  and  $S$  and choose **Edit | Merge Points.**]
10. Erase the traces. [Choose **Display | Erase Traces.**]

11. Split variable  $b$  from its line and merge it to variable  $f(a)$ . [Select  $b$  and choose **Edit | Split Point from Parallel Line.** Then select  $b$  and  $f(a)$  and choose **Edit | Merge Points.**]



- Q5 Which point in your sketch is now the input to function  $g$ ?
12. Fix the label of point  $g(b)$  to use the new name of  $g$ 's input. [Use the **Text** tool to change the label of  $g(b)$  to  $g(f(a))$ .]
- Q6 Why does this new name make sense? Explain, in terms of your construction, what  $g(f(a))$  means.
13. Select the middle axis, variable, and segments, make a Hide/Show button, and hide them. Construct and trace the segment from  $a$  to  $g(f(a))$ .
- Q7 Drag independent variable  $a$  and observe the behavior of  $g(f(a))$ . What can you say about the relative speed and direction of these two variables? Are there any fixed points? If so, where are they?

- Q8 Record your observations for each of the three functions in the table below. For Relative Rate of Change record whether the dependent variable moves in the same or opposite direction as the independent variable, and how much faster or slower it moves. For Fixed Point record whether or not there is a fixed point, and whether it's to the left or right of the center point of the axis.

Function	Variables	Relative Rate of Change	Fixed Point? Where?
$f$	$a$ and $f(a)$		
$g$	$b$ and $g(b)$		
$g \circ f$	$a$ and $g(f(a))$		

## COMPOSE MORE FUNCTIONS

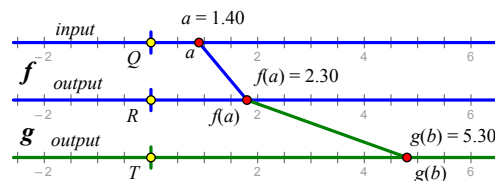
- Q9 Use pages 3, 4, and 5 of the sketch to practice composing more functions. For each page, make a table like the one above to record the relative rate of change and location of the fixed point (if any).

## MEASURE THE VARIABLES

Your observations concerning relative rate of change and locations of fixed points may not be very exact. To make them more precise, you can put numbers on the axes and measure the values of the variables.



13. Return to page 1, and turn each horizontal axis into a number line. [Press and hold the **Custom Tool** icon and choose the **Number Line** tool from the list that appears. Click the **Number Line** tool on points  $Q$ ,  $R$ , and  $T$  to turn their three axes into number lines.]
14. Measure the values of the three variables on the number line. [Press and hold the **Custom Tool** icon again, but this time choose the **Value on Parallel Line** tool. Click the **Value on Parallel Line** tool on the independent variable  $a$  and its number line, then on the “intermediate” variable  $f(a)$  and its line, and finally on the dependent variable  $g(f(a))$  and its line.]



- Q10 Use these numeric values to revise your table of relative rates of change and fixed points to make it more precise.

## EXPLORE MORE

On page 6, use the axes to make three interesting dynagraphs. Label the functions  $f$ ,  $g$ , and  $h$ . Then construct and label the composition of all three. Drag the independent variable to observe its behavior.