Cartesian Composition A Name:

In this activity you'll evaluate and compose functions using Cartesian graphs. By using the graphs directly, you don't need any equations for the functions.

EVALUATE TWO FUNCTIONS

1. Open **Cartesian Composition.gsp.** Page 1 contains a coordinate system and the graphs of functions f and g. Construct points a and b on the x-axis. [To create the points, use the **Point** tool. To label them, use the **Arrow** tool to select both points, and choose **Display | Label Points.**]

Q1 To evaluate $f(a)$, you can go vertically from a to the graph of f, and then horizontally from there to the y-axis. Explain why this works.

2. Construct a vertical line through a, find its intersection with f, and from the intersection construct a horizontal line. [For the vertical, select a and the x-axis and choose **Construct | Perpendicular.** For the intersection with f, click the intersection. For the horizontal line, select the intersection and the y-axis and choose **Construct | Perpendicular.**]

3. Construct the intersection where the horizontal line hits the y-axis, and label it $f(a)$. Also label the intersection on the graph $(a, f(a))$.

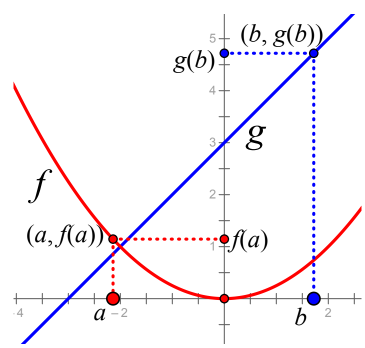
Q2 Why do these labels make sense for the points they label?

4. Hide the perpendicular lines, and construct dotted red segments from a to $(a, f(a))$ and from $(a, f(a))$ to $f(a)$. [Use the **Segment** tool to construct a segment, and choose **Display | Line Style | Dotted** to make it dotted.]

Q3 Vary a from left to right all the way along the x-axis, and describe the motion of $f(a)$. When does it move up? When does it move down? When does it move fastest, and when slowest? Where is its lowest point?

5. Starting from independent point b, construct points $(b, g(b))$ and $g(b)$ connected by dotted segments. Use the same steps as above (steps 2–4).

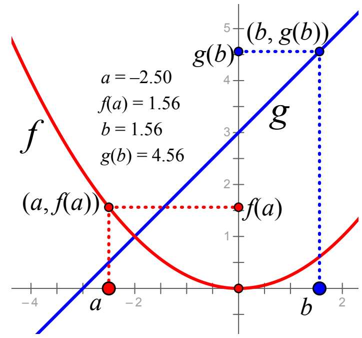
Q4 Vary b along the x-axis, and describe the motion of $g(b)$.



SET THE INPUT OF G TO THE OUTPUT OF F

6. To make it easier to drag precisely, measure the x-coordinates of a and b, and the y-coordinates of $f(a)$ and $g(b)$. Label the measured values to match the labels of the points. [Select points a and b and choose **Measure | Abscissas.** Select points $f(a)$ and $g(b)$ and choose **Measure | Ordinates.**]

Q5 Drag point a to –2.60. What is the value of $f(a)$?



Q6 To use the output of f as the input to g, drag b to get its value as close as you can to the value of $f(a)$. What is the resulting value of $g(b)$?

There are actually three ways to make $b = f(a)$: by hand, by action button, and by merging.

7. Drag a so that $a = 3.20$, and then drag b to match, so that $b = f(a)$.

Q7 When $a = 3.20$, what are the values of $f(a)$ and $g(b)$? What advantages and disadvantages do you notice for this method of dragging by hand?

You now have two points labeled $f(a)$, one on the y-axis and one on the x-axis. They should both have the same value.

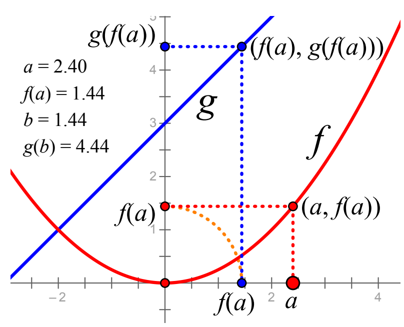
8. Vary a to make it –2.40.

9. To make a target for the action-button method, rotate point $f(a)$ by –90° around the origin. Label the rotated point $f(a)$. [Double-click the origin to mark it as the center. Then select point $f(a)$ on the y-axis and choose **Transform | Rotate.** Set the angle of rotation to –90° and press Rotate.]

10. To show the rotation as a dotted line, construct the arc between the two points labeled $f(a)$. [Select in order the origin, point $f(a)$ on the x-axis, and point $f(a)$ on the y-axis. Then choose **Construct | Arc on Circle.**]

11. Make a Movement button to move b to the new point $f(a)$. [Select in order point b and point $f(a)$ on the x-axis, and choose **Edit | Action Buttons | Movement.** Press OK to accept the default button properties.]

Q8 Press the new button. What is the resulting value of $g(b)$? Vary a and press the button again. What advantages and disadvantages do you notice for the action-button method of using the output of f as the input to g?



12. Vary a to make it 2.00. Split point b from its axis and merge it to point $f(a)$ on the x-axis. [After setting a to 2.00, select point b and choose **Edit | Split Point from Axis.** Then select both point b and the plotted point on the x-axis, and choose **Edit | Merge Points.**]

13. Point b is now labeled $f(a)$, so relabel measurement b to match, by changing it to $f(a)$. Similarly, change every other label that contains b by changing the b to $f(a)$.

Q9 Vary a to make it –1.80. What is the resulting value of $g(f(a))$? What advantages and disadvantages do you notice for this method?

14. Save your sketch so you can use it in Part B of this activity.

Cartesian Composition B Name:

In the second part of this activity you'll trace the composed function, create its graph, and then experiment with composing other functions.

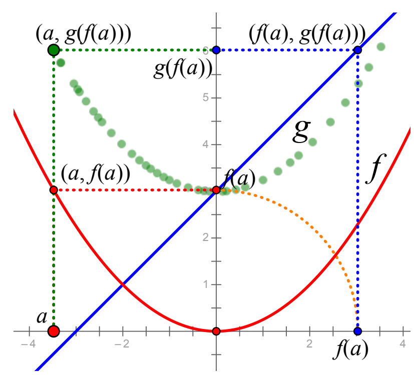
TRACE THE COMPOSITE FUNCTION

Q1 Fill in the blanks to describe the composition you created in Part A: Independent variable ___ on the ___ axis uses function ___ to produce intermediate variable ___, which uses function ___ to produce the dependent variable ______ on the ___ axis.

To trace the graph of the composite function, you need a point that uses the independent variable as its *x*-value and the dependent variable as its *y*-value.

1. Drag *a* to –3.60. Construct a vertical line through independent variable *a* and a horizontal line through dependent variable *g*(*f*(*a*)). [Select point *a* and the *x*-axis, and choose **Construct | Perpendicular Line.** Use the same method for the second perpendicular.]

2. Construct the intersection of the two lines, label it *(a, g(f(a)))*, color it green, and turn on tracing. [Click the **Arrow** tool on the intersection to construct it. Select the intersection and choose **Display | Trace Intersection** to trace it.]



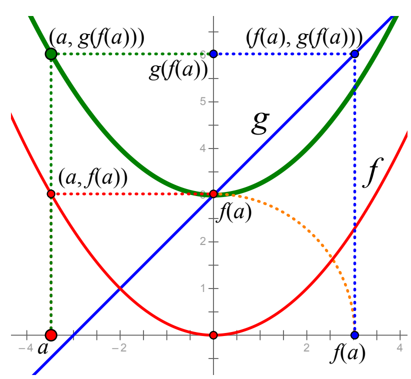
3. Hide the horizontal and vertical lines, and construct dotted green segments connecting *(a, g(f(a)))* to the axes.

4. Vary *a* along the *x*-axis to trace the composite function.

Q2 Describe the shape of the traces. How does this shape relate to the shapes of functions *f* and *g*?

CONSTRUCT THE GRAPH OF THE COMPOSITE FUNCTION

5. Turn off tracing for intersection (*a*, *g*(*f*(*a*))) and then erase the traces. [Select the intersection and uncheck the command **Display | Trace Intersection.** Then choose **Display | Erase Traces.**]



6. Construct the locus of intersection (*a*, *g*(*f*(*a*))) as *a* varies along the *x*-axis. [Select point *a* and the intersection, and choose **Construct | Locus.**]

Q3 Drag a along the x-axis. How does the motion of intersection $(a, g(f(a)))$ relate to the locus?

Q4 How does this locus give you an easier way to evaluate $g(f(a))$ given a?

ANIMATE THE COMPOSITION

You can make an animation that shows a point traveling along the dotted lines of your sketch, starting at independent variable a on the x-axis and ending at dependent variable $g(f(a))$ on the y-axis.

To use a custom tool, press and hold the Custom Tool icon and choose the tool from the list that appears.

7. Use the **Move Forward** custom tool on each dotted line or arc. Click the beginning point of the dotted line, then the dotted line itself, and finally the ending point. The result is a *Forward* button for each dotted line.

8. Combine these Forward buttons into a single presentation button. [Select the buttons in order and choose **Edit | Action Buttons | Presentation.** Make the presentation sequential.]

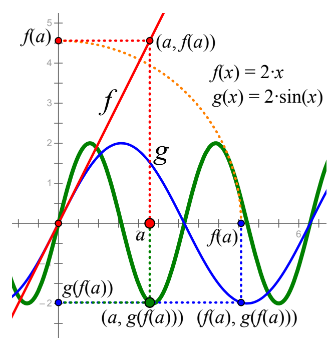
9. Press the button to reset all the points of the animation. After it finishes, press it again to see the animation.

Q5 The animation takes five stages: two segments, an arc, and two more segments. Describe the mathematical meaning of each stage.

EXPLORE DIFFERENT FUNCTIONS

10. Show the equations for functions f and g. [Click the Information tool on graph f, click on the "Function f" link in the balloon, and turn off the Hidden checkbox in the balloon for function f. Do the same for g.]

11. Edit the equations to try different combinations. For instance, you might try making $g(x) = x - 2$, or $g(x) = -x + 4$. What happens if you make $f(x) = \sin(x)$? Make up your own combinations to try.



Q6 Record the two most interesting combinations you tried; write down the equations, and draw the resulting graphs on your paper.

Q7 What discoveries did you make? Were you able to make a graph move up and down? Were you able to make a graph move left and right? Were you able to stretch a graph vertically, or to shrink it horizontally? Here's a hint: try making one of the functions more complicated (such as $\sin(x)$) while keeping the other one simpler (such as a linear function). Change the slope or intercept of the linear function, and you'll get some interesting results.

