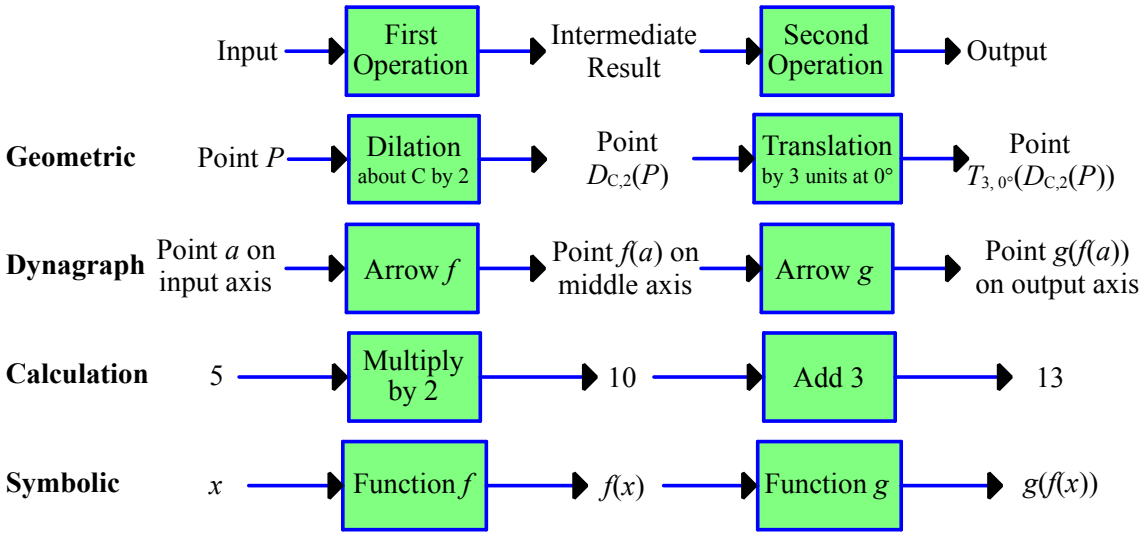
Calculation Composition 1 Name:

The basic idea behind function composition is that you start with an input, you perform some operation on it to get a result, and then you perform a second operation on that result, producing the output. The input and output values might be points, they might be numbers, or they might even be some other kind of object:



In Part 1 of this activity you'll compose two calculations, one after the other, to produce a numeric output value from any numeric input value, and you'll collect these values in a table. In part 2 you'll show the values as a dynagraph.

CREATE A VARIABLE AND TWO CALCULATIONS

1. Open **Calculation Composition.gsp,** and go to page 1, which is empty.
2. Create independent variable *a*. [Choose **Number | New Parameter,** type *a* for the Name, and type 3.0 for the Value.]

For more details, use **Help | Reference Center** to look up "keyboard adjustment."

Q1 With *a* selected, press the + sign on the keyboard several times. Then press the – sign. What happens when you press these keys?

3. Use "multiply by 2" for the first operation. [Choose **Number | Calculate,** type "2*," click variable *a* in the sketch, and press OK.]
4. Use "add 3" for the second operation. [Choose **Number | Calculate,** click the calculation $(2 \cdot a)$ in the sketch, and type "+3." Then press OK.]

$a = \boxed{5.0}$
$2 \cdot a = 10.0$
$(2 \cdot a) + 3 = 13.0$

Q2 Change *a* to 4.0. What are the other numbers? [In addition to pressing the + and – keys, you can edit *a* directly to change its value.]

Q3 Why do these calculations create a composite function? Explain.

CREATE A TABLE OF VALUES

5. Create a table using variables *a, 2a,* and *2a + 3.* [Select the three values in order and choose **Number | Tabulate.**]

6. Make a button to animate *a* between –5 and 5 discretely by 0.5 unit per second. Label it *Vary a Discretely.* [With *a* selected, choose **Edit | Action Buttons | Animation** and make the value change discretely by 0.5 units per 1 sec. from –5 to 5.]

7. Set *a* to 5.0 and add 20 data rows to the table as you change *a* to show all values between 5.0 and –5.0. [Select the table, choose **Number | Add Table Data,** and set the radio button to "Add 20 entries." Then press the animation button.]

a	2·*a*	(2·*a*) + 3
5.0	10.0	13.0
4.5	9.0	12.0
4.0	8.0	11.0
3.5	7.0	10.0
3.0	6.0	9.0

You can use the table of values to get two kinds of information. By looking at a single row, you can find the output value for a particular input value, and by looking at two rows, you can see by how much the input and output values changed between the two rows.

Q4 In this table, what is the value of the intermediate variable when $a = 1.5$?

Q5 What is the value of the dependent variable when $a = -2.0$?

Q6 By how much does the dependent variable change when the independent variable changes from 3.5 to 3.0? Also express your answer as unit rate of change. (Unit rate of change is the amount the output would change if the input changed by exactly one unit.)

Q7 By how much does the output change when the input changes from –1.0 to –3.0? Also express your answer as unit rate of change.

Q8 Change the first calculation from *2·a* to *3·a* and recreate the table data. What output corresponds to $a = -2.5$? What is the unit rate of change from $a = 0.5$ to $a = 1.0$? [To remove old data, select the table and choose **Number | Remove Table Data.** Add new data as you did in step 7.]

8. Save the sketch to use in the next activity, Calculation Composition 2.

EXPLORE MORE

Q9 Invent some interesting calculations of your own to experiment with. You can choose any calculation you like, as long as the first one depends on a, and the second one depends on the first. Report on the functions you created, list a set of corresponding values (independent, intermediate, and dependent values), and give the unit rate of change between two adjacent rows in your table.

Calculation Composition 2 Name:

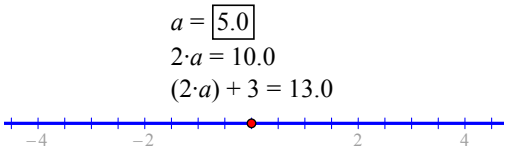
In Part 1 of this activity you collected values of a composed function in a table. In Part 2, you'll create a dynagraph that shows the same data as the table.

CREATE A DYNAGRAPH

1. Open your saved sketch from Part 1.
2. On page 1, copy the three values and the *Vary a Discretely* button. Paste them on page 2. [Select all four objects, choose **Edit | Copy,** go to page 2, and choose **Edit | Paste.**]

Page 2 also has an adjustable unit length and three points.

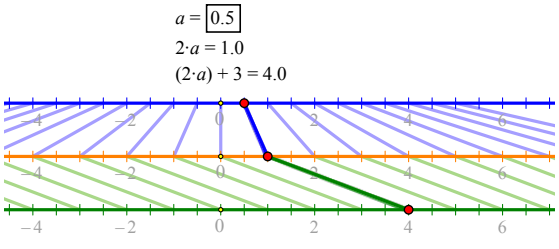
3. If you changed the original calculations, edit the copies on page 2 to make the first $2{\cdot}a$ and the second $(2{\cdot}a) + 3$. [Double-click to edit a calculation.]
4. Use the **Number Line** custom tool to create number lines through each of the three points using the given unit length. [Press and hold the **Custom Tool** icon and choose **Number Line** from the list of tools. Click the tool on each point, and then choose the **Arrow** tool.]



5. Plot the independent variable (a) on the top number line, the intermediate value ($2a$) on the middle number line, and the dependent variable ($2a + 3$) on the bottom number line. Label the points *a, 2a,* and *2a + 3*. [Select independent variable a and the top number line and choose **Graph | Plot Value on Axis.** Plot the others the same way.]
6. Connect the plotted points with traced segments. [Use the **Segment** tool to connect the points. Choose **Display | Trace Segment** to trace them.]
7. Press *Vary a Discretely* to vary the independent variable and trace the function's behavior.

Q1 Use the dynagraph to find value of the dependent variable when $a = 1.5$. Explain how you found this from the dynagraph.

Q2 Use the dynagraph to find the change in the output, and the unit rate of change when the input changes from –3.0 to –2.5. Explain your method.



CHANGE THE FUNCTION

By changing the calculations, you can change the functions you're composing.

8. Change the first calculation from $2 \cdot a$ to $a - 1$. [Double-click the calculation to edit it. Remove "2*" in front of *a* and insert "–1" after it.]
9. Change the second calculation to $(a - 1) / 2$. [Double-click the calculation and change "+3" to "/2".]

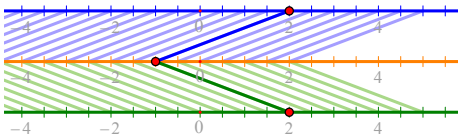
What difference does it make if a varies continuously instead of discretely? Check it out for yourself.

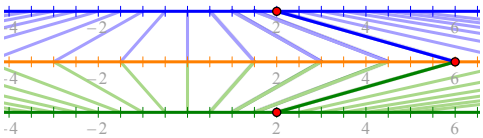
Q3 Press *Vary a Discretely* to vary the independent variable. Describe the shape of the dynagraph traces, and sketch them on your paper.

Q4 What values of the intermediate and dependent variables correspond to $a = 2.0$?

Q5 What is the change in the dependent variable when *a* changes from 3.0 to –1.0? Convert your result to a unit rate of change.

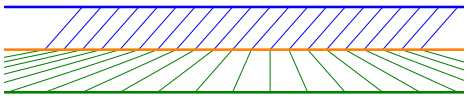
Q6 In each pattern below, the output is always equal to the input. Edit your calculations to produce these patterns. What calculations did you use?

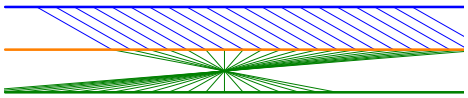
a. 

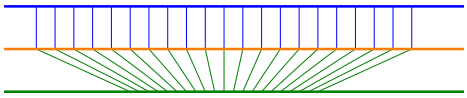
b. 

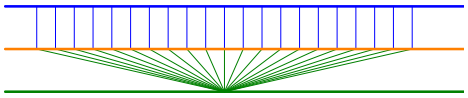
EXPLORE MORE

Q7 From each pattern of dynagraph traces shown here, how much can you tell about the two functions that produced it?

a. 

b. 

c. 

d. 

Q8 Make a Hide/Show button to hide the two connecting segments and the intermediate axis, construct a new segment to connect the independent and dependent variables, and make a Hide/Show button for the new segment. Then use the buttons to change between viewing the behavior of the original two functions and the behavior of the composed function.

Q9 Invent some interesting calculations of your own to experiment with. You can choose any calculation you like, as long as the first one depends on *a*, and the second one depends on the first. Report on the functions you created and the behavior you observed.